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Generation of coloured acoustic noise samples with non-Gaussian distributions

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Abstract: This study proposes a method for generating acoustic noise sequences with different distributions and coloured spectra. The noise samples are obtained by passing a non-Gaussian white noise through a finite impulse response filter. The resulting power spectral density is proportional to $1/f^{\beta}$, where β is a positive power-law exponent. The noise samples pattern is determined by the Kurtosis ratio. The proposed generator is evaluated by comparing real and artificial acoustic noises statistics in the time and frequency domains. The results show that the generated coloured sequences achieved the PSD decaying rate and also the non-Gaussian pattern of the real environmental acoustic noises.

1 Introduction

The presence of background noise can lead to severe performance degradation of applications and systems. The analytical solutions are generally based on the classical assumption of additive white Gaussian noises (WGN). However, in contrast with such a hypothesis, coloured spectra or $1/f^{\beta}$ noises have been detected in music [1], ocean [2], speech [3], optical devices [4], economic data [5] and a variety of other applications as listed in [6].

In the last decade, some techniques were proposed for modelling and generating coloured noises [5, 7–9]. Most of these approaches are based on filtering white noises to obtain target power spectral density (PSD) [7, 8, 10]. Although such techniques provide coloured PSD shape, they are limited in the sense that, except for the Gaussian cases, they do not guarantee the required probability density function (PDF) of the generated noise samples. In [2], a random number generator was proposed to obtain non-Gaussian coloured noises. However, this referred work did not include the definition of the filter to achieve the $1/f^{\beta}$ PSD. Other methods [11, 12] were proposed for generating random sequences with non-Gaussian distributions and coloured spectra. However, in [11] the target PSD is only guaranteed for a very large sequence length, whereas in [12] the noise samples are restricted to infinitely divisible PDFs.

This paper proposes a generation method for artificial acoustic noises with different distributions and coloured spectra. To attain the $1/f^{\beta}$ spectra, the noise samples are obtained by filtering a white sequence with a discrete time fractional order integrator proposed by Al-Alaoui [13]. The coloured noise samples distribution is determined by the Kurtosis ratio (K) [14]. The target value of K is obtained by properly choosing the pattern of the white noise in the filter input. This method assures that the noise samples reproduce the first, second and fourth moments of the real acoustic noises. Moreover, there is no restriction on the size of the generated noise sequences. Although this work refers to non-Gaussian coloured spectra noises, Gaussian pattern and white PSD can also be generated using the proposed approach.

For the evaluation of this proposal, coloured noise sequences of different sizes are generated with the same values of K and β of three different real acoustic noises. The PDF and PSD curves of the real and the artificial noises are presented for comparison. The study on the effects of the total number of filter coefficients is also considered in this work. The results show that both Gaussian and non-Gaussian distributions of the real noises, and also their coloured spectra, can be represented by the generated noise samples.

The main contributions of this work are:

- It defines a finite impulse response (FIR) filter to achieve the $1/f^{\beta}$ PSD shape of the noise sample sequence. The coefficients are calculated by expanding the target transfer function in power series, and they determine the relationship between the Kurtosis ratios of filter input and output.
- It determines the minimum number of filter coefficients to achieve coloured spectra. It is shown that, for sequences with more than 5×10^{4} samples, filters with less than 2000 coefficients result in oscillating PSDs. On the other hand, the frequency response of filters with more than 2000 coefficients are quite similar.
- It proposes a simple and fast estimation method for the PSD exponent. The results empirically determined the upper bound $\hat{\beta} \simeq 1.85$ for the estimated PSD exponent of the artificially generated noises.

This work is organised as follows. Section 2 presents the proposed non-Gaussian coloured noise samples generation. This includes the generation of the non-Gaussian white noises and the calculation of the filter coefficients. In Section 3, the coloured noise generator is evaluated by reproducing three environmental acoustic noises with different distributions and coloured spectra. A comparison among real and artificial noises is also presented in this section. Finally, the conclusions of this work are discussed in Section 4.

2 Non-Gaussian coloured noise generator

The PSD ($S(f)$) of a noise sample sequence can be approximated by

$$S(f) = c\frac{1}{f^{\beta}} \tag{1}$$

where c is a positive real number, and β is in the range $0 \le \beta \le 2$. Depending on the PSD decaying rate, noises can be classified as white ($\beta \simeq 0$), pink ($\beta \simeq 1$) and brown ($\beta \simeq 2$).

The classical approach of filtering a white noise to obtain the coloured spectra noises is illustrated in Fig. 1. To achieve the PSD shape in (1), the frequency response of the filter should be proportional to $1/f^{\beta/2}$. In this paper, a FIR filter is used for this purpose, and the filter coefficients are calculated based on the Al-Alaoui digital integrator transfer function [13]. The white noise is represented by a sequence of statistically independent random numbers $\{X_m\}$.

The Kurtosis ratio is defined as the ratio of the fourth to the square of the second central moments. It is considered a useful statistic to determine whether the PDF of a sample sequence differs from the Gaussian pattern. Background noises can have Kurtosis different from a Gaussian distribution ($K = 3$). In [15], for example, the author measured different ocean noises with Kurtosis varying in the range $2.30 \le K \le 3.67$. The Kurtosis ratio of the white noise $\{X_m\}$ is given by

$$K_{X_m} = \frac{E[(X_m - m_{X_m})^4]}{\sigma^4_{X_m}} \tag{2}$$

where m_{X_m} and σ_{X_m} are the mean and the standard deviation of $\{X_m\}$, respectively. In order to represent Gaussian and non-Gaussian white noises, the sequence $\{X_m\}$ is generated as proposed in [2].

2.1 Non-Gaussian white noise samples generation

Consider a sequence of independent random numbers $\{W_m\}$, uniformly distributed in the interval $0 < W_m \le 1$. Each term of the sequence $\{X_m\}$ is obtained by

$$X_m = \left[\log\frac{1}{W_{2m-1}}\right]^n \sin(2\pi W_{2m}) \tag{3}$$

where n is a non-negative real number.

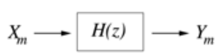

Fig. 1 *White noise $\{X_m\}$ is filtered to obtain an output sequence $\{Y_m\}$ with $1/f^{\beta}$ PSD*

If $n = 1/2$, the transformation in (3) leads to a Gaussian-distributed sequence. For any other value of n, the sample sequence $\{X_m\}$ has a non-Gaussian pattern. Although the PDF of $\{X_m\}$ cannot be explicitly determined for any values of n, its Kurtosis ratio is related with n according to

$$K_{X_m} = \frac{3}{2}\frac{\Gamma(4n+1)}{[\Gamma(2n+1)]^2} \tag{4}$$

where the gamma function $\Gamma(\cdot)$ is defined by

$$\Gamma(r) = \int_0^{\infty} t^{r-1}\mathrm{e}^{-t}\,\mathrm{d}t \tag{5}$$

This means that the desired Kurtosis ratio of $\{X_m\}$ can be obtained by choosing a value for n according to (4) and (5). Since the terms of $\{X_m\}$ are statistically independent, its PSD shape is $S_{X_m}(f) \propto 1/f^0$, and can be used to represent a white noise.

2.2 Calculation of the filter coefficients

The PSD of the output sample sequence $\{Y_m\}$, resulting from the filtering illustrated in Fig. 1, is given by

$$S_{Y_m}(f) = \sigma^2_{X_m}|H(\mathrm{e}^{\mathrm{j}2\pi fT})|^2 \tag{6}$$

where T is the sampling period and $|H(\mathrm{e}^{\mathrm{j}2\pi fT})|$ is the frequency response of the filter $H(z)$.

The Al-Alaoui rule [13] is adopted in the filter transfer function $H(z)$, with $\beta/2$ as the fractional order exponent

$$H(z) = \left[\frac{7T}{8}\frac{(1+z^{-1}/7)}{(1-z^{-1})}\right]^{\beta/2}, \quad 0 \le \beta/2 \le 1 \tag{7}$$

From (7), the amplitude frequency response of $H(z)$ is given by

$$|H(\mathrm{e}^{\mathrm{j}2\pi fT})| = \left[\frac{7T}{8}\right]^{\beta/2}\left[\frac{\sqrt{(50/49)+(2/7)\cos(2\pi fT)}}{2\sin(\pi fT)}\right]^{\beta/2} \tag{8}$$

which means that $S_Y(f) \propto 1/f^{\beta}$ as $f \to 0$.

Since in this paper $H(z)$ is assumed to be a FIR filter, its coefficients are calculated by a power series expansion (PSE) of (7)

$$H(z) = h(0) + h(1)z^{-1} + h(2)z^{-2} + \cdots + h(N-1)z^{-(N-1)} \tag{9}$$

The filter coefficients are obtained by the convolution [8]

$$h(k) = \left[\frac{7T}{8}\right]^{\beta/2} a(k) * b(k) \tag{10}$$

where $a(k)$ and $b(k)$ are the first $N/2$ coefficients given by the PSE of the numerator and the denominator of (7), respectively,

$$[1+z^{-1}/7]^{\beta/2} = a(0)+a(1)z^{-1}+a(2)z^{-2} + \cdots + a(N/2-1)z^{-(N/2-1)} \quad (11)$$

$$[1-z^{-1}]^{-\beta/2} = b(0)+b(1)z^{-1}+b(2)z^{-2} + \cdots + b(N/2-1)z^{-(N/2-1)} \quad (12)$$

Hence, the coefficients can be expressed by the following recurrences

$$a(k) = \frac{\beta/2-k+1}{7k}a(k-1), \quad a(0)=1 \quad (13)$$

$$b(k) = \frac{\beta/2+k-1}{k}b(k-1), \quad b(0)=1 \quad (14)$$

The coloured noise samples are determined by the convolution

$$Y_m = X_m * h(m) = \sum_{k=0}^{N-1} h(k)X_{m-k} \quad (15)$$

If the sequence $\{X_m\}$ is Gaussian distributed, the coloured noise samples determined by (15) also have a Gaussian pattern. When $\{X_m\}$ is non-Gaussian, $\{Y_m\}$ does not have the same distribution as $\{X_m\}$. In such cases, the distribution of $\{Y_m\}$ cannot be expressed in a closed form. However, its Kurtosis ratio (K_{Y_m}) is related to the filter coefficients and to the Kurtosis ratio (K_{X_m}) of the input sequence by the following [2]

$$K_{Y_m} = 3 + (K_{X_m}-3)\frac{\sum_{k=0}^{N-1} h(k)^4}{\left(\sum_{k=0}^{N-1} h(k)^2\right)^2} \quad (16)$$

Thus, given the filter coefficients and a target value of K_{Y_m}, the value of K_{X_m} is obtained by solving (16).

2.3 Proposed coloured noise generation method

In order to obtain the output sequence $\{Y_m\}$ with the target values of β and K_{Y_m}, the filter coefficients and the terms of $\{X_m\}$ must be calculated as described in Sections 2.1 and 2.2. According to (3)–(16), the proposed method for generating the coloured noise samples is divided into six consecutive steps

1. Considering the required value of β, calculate the filter coefficients $h(k)$ using (10), (13) and (14);
2. Given the filter coefficients and the target value of the Kurtosis ratio K_{Y_m}, determine K_{X_m} in (16);
3. Determine the value of n in (4) using the value of K_{X_m} obtained in step 2;
4. Generate the noise sample sequence $\{X_m\}$ as determined in (3);
5. Obtain the sequence $\{Y_m\}$ with the corresponding coloured PSD by filtering the white noise, according to (15);
6. Estimate the values of $\hat{K}_{Y_m}$ and $\hat{\beta}$ of $\{Y_m\}$. If the deviations between the estimated ($\hat{K}_{Y_m}$, $\hat{\beta}$) and the target values (K_{Y_m}, β) are above some threshold, return to step 4. In this work, the threshold is set to a maximum error of 1%. However, this error is not restricted to this value.

In step 6, the estimation of the Kurtosis ratio $\hat{K}_{Y_m}$ is carried out using (2), exchanging $\{X_m\}$ for $\{Y_m\}$. For the $\hat{\beta}$ estimation, the linearisation of (1) is proposed, using the logarithmic function. A linear regression is applied on the resulting data, expressed by

$$\ln S(f) = \ln c - \beta \ln f \quad (17)$$

The value of $\hat{\beta}$ is then estimated by the slope (with exchanged signal) of the obtained regression line.

After obtaining the coloured noise samples $\{Y_m\}$, the transformation

$$Y_{m'} = pY_m + q, \quad p, q \in \mathbb{R} \quad (18)$$

is used to obtain a new sequence $\{Y_{m'}\}$ with the target values of mean and variance determined by chosing p and q. The Kurtosis and the PSD shape of $\{Y_{m'}\}$ are the same as $\{Y_m\}$.

3 Experiments and results

This section presents the results obtained in the experiments conducted to evaluate the proposed noise generation method. The main issue is to validate the non-Gaussian coloured sequences on representing the acoustic environmental noises. For this purpose, three real noises, acquired from different acoustic sources, are used as reference: Airplane, Factory and Volvo. These noises were collected from the NOISEX-92 database [16], and they have a duration of 235 seconds and a sampling rate of 19.98 kHz. The estimation results of the Kurtosis ratio and the value of $\hat{\beta}$ are presented in Table 1. Each noise has about 4.7 M samples, which leads to a precision error of 0.000532 considering a confidence degree of 99%, using the Chebyshev inequality.

It follows from (8) that the PSD shape of the generated noise is not the same as expressed in (1) for values of f close to the Nyquist frequency ($f \to 1/2T$) [8]. This behaviour leads to a non-linear relationship between the values of β, adopted in the noise generator equations, and the values of $\hat{\beta}$ that are estimated from the generated sample sequences. For the evaluation of this non-linearity, a set of 41 coloured sequences are generated with values of β varying from 0 to 2, with steps of 0.05.

The values of $\hat{\beta}$, estimated from each of the obtained coloured sequences, are illustrated in Fig. 2. As can be noted, the PSD decaying rates of the generated coloured noises, are restricted to $0 \le \hat{\beta} \le 1.85$. In order to generate coloured spectra noises with the target $1/f^{\hat{\beta}}$ PSD, an eighth degree polynomial is used as an approximation to the curve

Table 1 Kurtosis ratios (K) and PSD exponents (β) of the real acoustic noises collected from the NOISEX-92 database

Noises	Airplane	Factory	Volvo
$\hat{\beta}$	1.19	1.83	1.86
$\hat{K}$	2.96	3.36	3.54

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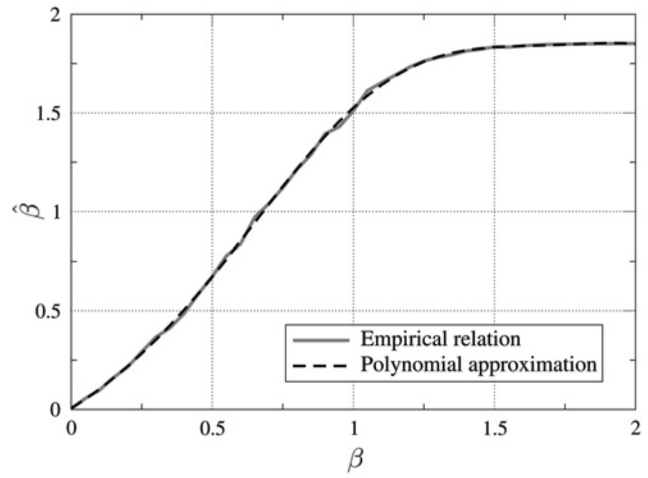

Fig. 2 *Relationship between the values of β adopted in the generation method and the corresponding estimated values $\hat{\beta}$ of the obtained sequences*

in Fig. 2,

$$\hat{\beta} = 0.0060819 + 0.85554\beta + 1.3641\beta^2 - 2.6467\beta^3 + 7.269\beta^4 - 9.9295\beta^5 + 6.2208\beta^6 - 1.8132\beta^7 + 0.20016\beta^8 \quad (19)$$

Thus, the value of β to be used in step (1) of the generation method must be adapted by solving (19). Such a solution is considered in all the experiments presented in this work.

A second set of experiments is conducted to study the effects of the total number of filter coefficients (N) adopted in (9). For this purpose, artificial coloured noises are generated considering different values of N. The sample sequences are generated with the same duration, sampling rate, Kurtosis ratio and PSD decaying rate of the real Volvo noise.

Fig. 3 shows the PSDs of the generated coloured noises, obtained with the different number of filter coefficients. From the bottom to the top, the PSD curves are shifted 6 dB from each other for the sake of visualisation. As can be noted, the PSD shapes oscillate for $N < 2000$, mainly at low frequencies. On the other hand, PSDs obtained with $N > 2000$ are considerably similar. This means that, to avoid the PSD fluctuations, the calculation of each sample Y_m requires at least 2000 terms in the summation of (15). The number of arithmetic operations per output sample can be reduced using a deterministic signal modelling technique [8, 17] which approximates the PSE in (9) by a rational function. This approximation leads to the implementation of an infinite impulse response filter, which would change the relation between K_{Y_m} and K_{X_m}, expressed in (16). Thus, the use of such a technique is not appropriate for the proposed solution.

Table 2 Kurtosis ratios ($\hat{K}$) and PSD exponents ($\hat{\beta}$) estimation results of generated coloured noise signals

Noises	Airplane	Factory	Volvo
$\hat{\beta}$ (CI = 95%)	1.187 ± 0.042	1.828 ± 0.010	1.850 ± 0.003
$\hat{K}$ (CI = 95%)	2.958 ± 0.015	3.353 ± 0.016	3.532 ± 0.006
$\hat{\beta}$ (CI = 99%)	1.187 ± 0.024	1.828 ± 0.018	1.850 ± 0.005
$\hat{K}$ (CI = 99%)	2.958 ± 0.008	3.353 ± 0.028	3.532 ± 0.010

In a third set of experiments, coloured spectra noises are generated according to the three environmental acoustic noises collected from the NOISEX-92 database (Table 1). Five independent artificial noise sequences are generated with 5×10^4, 1×10^5, 5×10^5, 1×10^6 and 4.7×10^6 samples with the same Kurtosis ratio and β values of the real noises. To avoid the oscillating behaviour of the PSD shapes (Fig. 3), $N = 2000$ filter coefficients are adopted. The accuracies of the Kurtosis ratio and the β estimations are evaluated by the t-student method considering 95 and 99% confidence intervals (CI). The estimation results are presented in Table 2. These results show that the coloured noise samples, considering different sequence sizes, have values of $\hat{K}$ and $\hat{\beta}$ close to those determined by the real acoustic noises.

Figs. 4 and 5 depict the PSD and the PDF curves of the artificial and real acoustic noises obtained with 4.7×10^6 samples, respectively. As can be noted from Fig. 4, the PSD decaying rates of the artificial noises are quite similar to those presented by the real noises. These results reinforce the values estimated for the PSD exponent (Table 2). Fig. 4

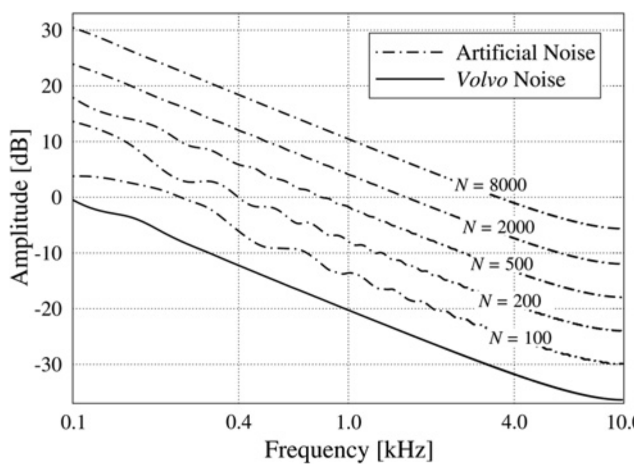

Fig. 3 *Comparison among the PSD of artificial noises for different numbers of filter coefficients*

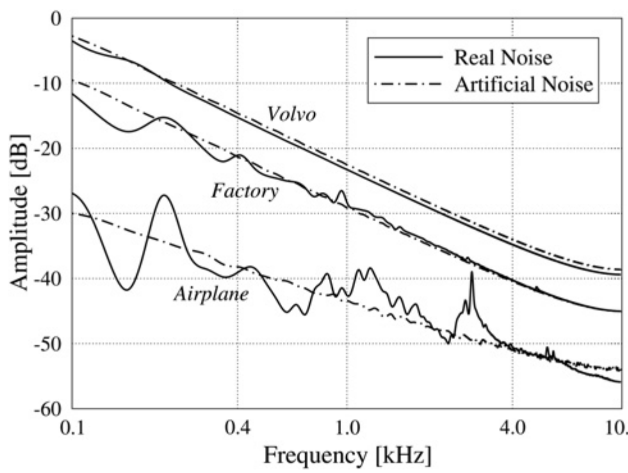

Fig. 4 *PSD of the real and generated acoustic noises*

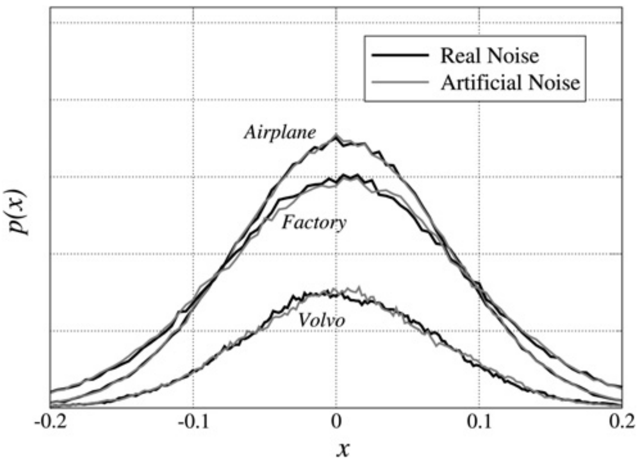

Fig. 5 *PDF of the real and generated acoustic noises*

Table 3 KL and Bh distances between generated and real acoustic noises

Noises	Bh	KL
airplane	2.18×10^{-4}	3.49×10^{-5}
factory	5.60×10^{-4}	8.28×10^{-5}
volvo	2.94×10^{-3}	4.66×10^{-4}

also demonstrates that the PSDs of the artificial noises do not follow the $1/f^{\beta}$ behaviour for large values of f ($f > 4.0$ kHz). This deviation is more easily noted for the Volvo and Factory noises, which present the highest PSD decaying rates. Thus, the differences between the values of β adopted in the generation method and the corresponding estimated values $\hat{\beta}$ are expected to increase with β. This explains the non-linear relation depicted in Fig. 2.

From Fig. 5, it can be seen that the distributions of the artificial noises are considerably close to the PDFs of the corresponding real noises. Note that even the non-Gaussian distributions (Factory and Volvo) were well represented by the generated noise samples. It should be noted that only the Kurtosis ratios and their first and second moments were used to obtain the artificial noises pattern.

The results of the Bhattacharyya (Bh) [18] and the Kullback–Leibler (KL) [19] distances, considering the distributions of the real and artificial noises (see Fig. 5), are shown in Table 3. The Bh and the KL distances between two PDFs $p_1(x)$ and $p_2(x)$ are defined by $\mathrm{Bh}(p_1, p_2) = -\log \int_{\mathbb{R}} \sqrt{p_1(x)p_2(x)}\,\mathrm{d}x$ and $\mathrm{KL}(p_1\|p_2) = \int_{\mathbb{R}} p_1(x)\log(p_1(x)/p_2(x))\,\mathrm{d}x$, respectively. Owing to its non-symmetrical nature, the KL values in Table 3 are calculated as the sum $\mathrm{KL}(p_1\|p_2) + \mathrm{KL}(p_2\|p_1)$. Note that, in accordance with Fig. 5, the lowest values for both distances were achieved for the Airplane noise. Hence, Fig. 5 and Table 3 reinforce that the class of PDFs obtained by the proposed generator are very interesting to represent the non-Gaussian distributions of environmental acoustic noises.

4 Conclusion

This paper proposes a method for non-Gaussian coloured spectra noises generation. The noise sample sequences are obtained by filtering a white noise using a discrete time FIR filter. The main contribution of this proposal is that the obtained $1/f^{\beta}$ PSD shape is determined together with a class of possible distributions, defined by its Kurtosis ratio. Three real environmental acoustic noises, with coloured PSDs and different PDFs, are used for the proposed evaluation of the proposed generator. The experimental results show that the generated artificial noise samples can represent both the PSD decaying rate and the distribution of the real acoustic noises.

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