Low-frequency optical noise generator using fractional statistics

L. Zão and R. Coelho

A design for a low-frequency $1/f$ optical noise generator using fractional statistics is proposed and demonstrated for the first time. The $1/f$ noise samples were obtained by transformation functions performed on pseudorandom binary sequences that modulate a laser source. The $1/f$ spectrum representation of the optical noise samples shows that this proposition is very promising for the investigation of this noise effect in optical devices and system reliability. It can also be useful for the spectral distribution measurements of ultra-narrow linewidth lasers.

Introduction: Low-frequency or $1/f$ noise has been widely observed in many optical devices including laser diodes [1], semiconductor lasers [2–4] and also in coherent communications systems [5]. The presence of $1/f$ noise is described by different factors such as device material characteristics, fluctuations in the number of carriers and also the quantum properties of photons. Thus, the investigation of the $1/f$ noise effect in optical devices and lightwave communications systems has become a key issue.

Noises are random processes described by the shape of their power spectral density (PSD, $S(f)$). The PSD of $1/f$ noises [6, 7] is defined by $S(f) = 1/f^{\beta}$ with $0 \le \beta \le 2$. $1/f$ fractional noise has $S(f) = f^{1-2H}$, where $1/2 < H < 1$ is the Hurst parameter [8]. The H parameter is also defined by the slow-decaying rate of the autocorrelation function (ACF, $\rho(k)$) of the noise samples. It represents the low-frequency or scaling invariance degree of the fractional noises and it is frequently close to 1. Mandelbrot and Van Ness [7] also showed that the $1/f$ noise statistics can be accurately represented by the fractional Brownian motion (fBm) random process. It has independent increments, indexed by the single scalar H parameter defined in $\Re$ with zero mean and continuous sample path (null at origin). Therefore, an optical $1/f$ noise generator based on fBm statistics is proposed and demonstrated in this Letter. The $1/f$ spectral behaviour is obtained from the ACFs of the noise samples generated with the fBm pattern. The $1/f$ fBm noise sample generation is based on transformation functions performed on uniform pseudorandom binary sequences. These functions are defined by the successive random addition algorithm [9] using the midpoint displacement (SRMD) technique [9]. Since all-optical logic and photonic integrated circuits are still very limited, the $1/f$ noise sample generation was performed on a high-speed field-programmable gate array (FPGA) development kit. Each noise level output is further applied to modulate a laser source to produce the optical $1/f$ noise sample.

1/f fractional Brownian noise and SRMD technique: For any instant $t > 0$, $X_H(t)$ is a fractional random function with Gaussian independent increments [7]; $X_H(0) = 0$, $E[X_H(t)] = 0$ and variance $|t_2 - t_1|$. For any τ and $r > 0$, it follows that $[X_H(t+\tau) - X_H(t)_{r\le 0}] \overset{d}{\simeq} r^{-H}[X_H(t+r\tau) - X_H(t)_{\tau\le 0}]$ where r is the scaling factor and $\overset{d}{\simeq}$ means similar in distribution. Thus, $X_H(t)$ is a self-similar random process, i.e. its statistical characteristics hold for any time scale. Moreover, the $X_H(t)$ are completely characterised by their mean (null), variance (σ^2) and H parameter. Considering a time index t defined at the interval [0, 1], the SRMD algorithm establishes that by setting $X(0) = 0$ and $X(1)$ as a Gaussian random variable (RV) with zero-mean and variance σ^2, $Var[X(1) - X(0)] = \sigma^2$ and $Var[X(t_2) - X(t_1)] = |t_2 - t_1|\sigma^2$ for $0 \le t_1 \le t_2 \le 1$. To achieve this property a random offset displacement (D_i) with zero-mean and variance δ_i^2, must be added to the noise sample where $\delta_i^2 = 1/2^{-(i+1)}\sigma^2$. For example, the $X(1/2)$ value is obtained by the interpolation of $X(0)$ and $X(1)$ with variance $\delta^2/2^{2H+1}$. Several iterations are then proceeded to compose a $1/f$ fBm random sequence. To achieve stationary increments, after the midpoint interpolation, a D_i of a certain variance ($\propto (r^n)^{2H}$; where r is the scaling factor) is applied to all points (time increments) and not just the midpoints. The maximum number of iterations is defined by $N = 2^{maxlevel}$ where *maxlevel* is defined in the interval [0,16]. The other SRMD inputs are the standard-deviation (*sigma*) and the H parameter.

Experimental setup and results: Fig. 1 shows the experimental setup used for the demonstration of the proposed $1/f$ fBm optical noise generator. The solution presented for the SRMD algorithm to generate the $1/f$ fractional noise samples was implemented in a high-speed field-programmable gate array (FPGA). The distributed-feedback (DFB) laser source with continuous-wave (CW) operation is directly modulated by a lithium niobate Mach-Zehnder modulator (MZM). The MZM is driven by the FPGA output at 150 million samples (electrical noise levels) per second. Polarisation controllers (PCs) are also inserted at the input and output of the MZM. The $1/f$ noise power spectral densities are evaluated after photodectection (*pin* receiver).

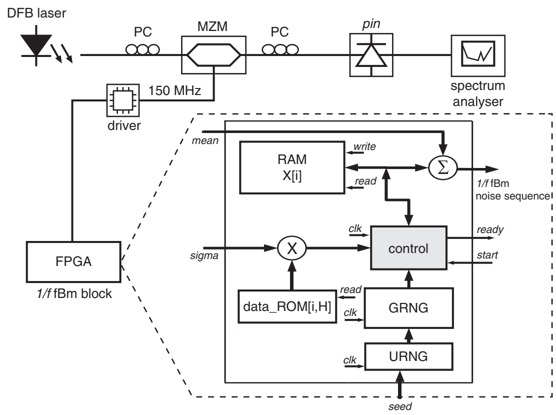

Fig. 1 *1/f optical noise generator: experimental setup*

The $1/f$ fBm noise generator is composed of the following main blocks: uniform random number generator (URNG); Gaussian random number generator (GRNG); data_ROM[i, H]; control and $X[i]$ RAM memory. The generation of Gaussian RVs is based on transformation functions performed on uniform variables using the Box-Muller method [10]. Note that the GRNG block [11] provides the mean value for the *Gauss(.)* function. It also generates the white Gaussian noise samples used in the experiments. The d, D and *level* are counter variables. The $X[i]$ has $i = 2^{maxlevel}$ increments or noise levels. The URNG block was coded to produce 32-bit uniformly distributed samples with periodicity 10^{10}. The linear feedback shift registers (LSFRs) are started by *seed* values to produce the pseudorandom sequences. The data ROM block performs the computation of the SRMD algorithm. The data_ROM is indexed by i and H and it is defined by $data\_ROM[i,H] := (delta[i]/sigma) = (1/2)^{iH}\sqrt{1/2}\sqrt{1-2^{2H-2}}$. The $delta[i]$ values are stored and addressed by the i and H indexes. This would be prohibitive owing to the large amount of memory resources needed for storing a wide range of $delta[i]$ values. However, H values can represented with only two digits after the decimal point. Thus, 1000 H values are necessary for each iteration of the SRMD algorithm. Since $0 \le maxlevel \le 16$, $delta[i]$ vector can have a maximum of 16 000 elements. Hence, 1.6% of the ROM memory resource was needed to store the $delta[i]$ vector. In fact, $1/f$ noise has $1/2 < H < 1$ (close to 1) and hence the memory needs can be reduced. This second memory block is used to store the output sample vector ($X[i]$). The binary representation of each $X[i]$ noise output was truncated to 16-bit wide. For example, for $maxlevel = 16$ it will be necessary to have $16 * 2^{16} \simeq 1.0$ Mbits of a RAM resource. The main functions of the control block are: read the GRNG block output (Gaussian samples); read the data ROM values according to the selected values indexed by i and H; evaluate the *delta* by multiplying previous data_ROM data to *sigma*; fill the initial values of the $X[i]$ vector with the computed $sigma * Gauss$ values; perform the *loops* iterations (one *while* and two *fors*); update the d, D and *level* counters; read fBm output noise sample levels from $X[i]$ vector. The $1/f$ noise SRMD block design implementation required approximately: 25 000 logic elements, four RAM blocks of 500 kbits each, ROM of 1 Mbits, 10 digital signal processor (*dsp*) blocks and six phase-locked loops (PLLs) used for clock generation to achieve the target output rate. The *multstyle* attribute was set for using the *dsp* blocks. Altera's Stratix devices have a fixed number of *dsp* blocks, which includes a fixed number of embedded multipliers. The *multstyle* attribute specifies the implementation style for multiplication operations (*) in the very high-speed integrated hardware description language (VHDL) source code. The implementation achieved 7.4% of logic elements, 52% of RAM memory and 1.6% of ROM memory for the longest $1/f$ fBm noise sequence.

Table 1 presents the H parameter estimation results obtained from the white and $1/f\,(H = 0.9)$ noise generated sequences. For this estimation the wavelet-based method [12] was used with 12 Daubechies filters and the 4–12 scale ranges. Five independent $1/f$ fBm noise sequences were generated with 10^6, 10^9, 10^{10}, 10^{11}, 10^{12} size. The accuracy of the H parameter estimation was evaluated with the *t-student* method considering 95 and 99% confidence intervals (CI). It can be seen that the $1/f$ noise generator achieved the low-frequency statistics with the H target values. As expected, the white Gaussian noise samples presented $H \simeq 1/2$ or β close to zero.

Table 1: H parameter estimation results

Noises	White	$1/f\,(H = 0.9)$
H (CI = 95%)	0.49 ± 0.00001	0.91 ± 0.00009
H (CI = 99%)	0.49 ± 0.00002	0.90 ± 0.00017

Fig. 2 shows the ACF plots obtained at the FPGA $1/f$ fBm noise block output. These results demonstrate the slow-decaying ($\Sigma_{k=-\infty}^{\infty}\rho(k) = \infty$) and exponential behaviour of the $1/f$ and white noises, respectively. The power spectral densities of the generated $1/f$ and white noise samples are shown in Fig. 3. These PSDs were measured using a high-performance 300 MHz bandwidth spectrum analyser. It can be seen that the $S(f)$ of the white noise is flat while for the $1/f$ optical noise samples it is concentrated in the low-frequency spectrum.

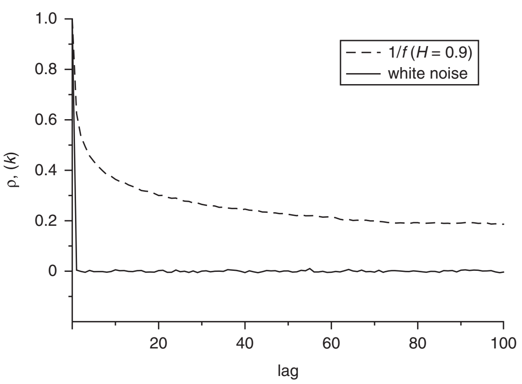

Fig. 2 *Autocorrelation function for white and 1/f noise*

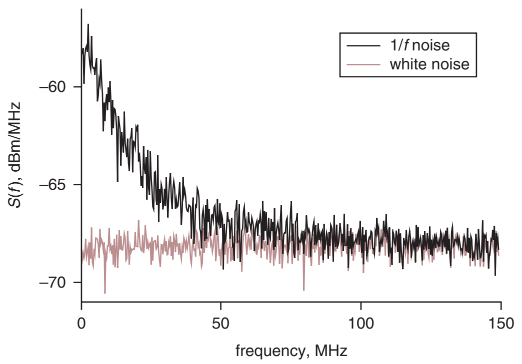

Fig. 3 *Power spectral density results for white and 1/f optical noise*

Conclusion: A design of a $1/f$ optical noise generator based on fractional statistics is proposed and demonstrated for the first time. The results showed that $1/f$ spectral behaviour is achieved by the optical fractional noise samples.

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One or more of the Figures in this Letter are available in colour online.

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